2024

(December)

FYUP : 3rd Semester Examination

MAJOR

PHYSICS

(Mathematical Physics—II and Experimental Physics—III)

PHY–200

Marks : 56

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer *any* **eight** questions

1. *(a)* Obtain the relation between Cartesian coordinates (x, y, z) and its spherical polar coordinates (r, θ, ϕ). 4

(b) Calculate the spherical coordinates of the point $P(3, 4, 5)$. 3

2. *(a)* Convert the complex number $z = 1 + i\sqrt{3}$ to its polar form. 2

(b) Find the Taylor series expansion of the function $f(x) = e^x$ about $x = 0$. Write down the first four terms of the series. 5

3. (a) Derive the Cauchy-Riemann conditions for an analytic function. 5

(b) The real and imaginary parts of $f(z)$ are $u(x, y) = x^2 - y^2$ and $v(x, y) = 2xy$. Determine whether $f(z)$ is analytic at any point. 2

4. (a) State and prove Cauchy's integral formula. 5

(b) Using the above formula, evaluate $\oint_C \frac{e^z}{z-1} dz$, where $|z| = 2$. 2

5. Write down the Legendre's differential equation. Derive the Rodrigue's formula for Legendre's polynomials $P_n(x)$. Find $P_2(x)$. 1+4+2=7

6. Solve the equation

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

where α is the thermal diffusivity constant with the following conditions :

Boundary conditions : $T(0, t) = 0$
$T(L, t) = 0$

Initial conditions : $T(x, 0) = f(x)$ 7

7. (a) Prove that the Hermite polynomials $H_n(x)$ satisfy the recurrence relation

$$H_{n+1}(x) = 2xH_n(x) - 2xH_{n-1}(x)$$ 4

(b) If $H_0(x) = 1$ and $H_1(x) = 2x$, find $H_2(x)$ and $H_3(x)$. 1½+1½=3

8. (a) Write down the general form of a second-order linear differential equation. Find the conditions for ordinary, regular and irregular singular points. 1+3=4

(b) Examine whether $x = 0$ a regular singular or an irregular singular point of the differential equation

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - (x^2 + 1)y = 0$$ 3

9. (a) Derive the expression for the Legendre's duplication formula. 5

(b) Evaluate $\Gamma(-\frac{5}{2})$. 2

10. (a) Define beta function $\beta(m, n)$ and gamma function $\Gamma(n)$. Write down the relationship between them. 1+1+1=3

(b) Evaluate $\beta(2, 3)$. 4

11. Find the eigenvalues and its corresponding eigenvectors of the matrix

$$M = \begin{bmatrix} 5 & -2 \\ -2 & 2 \end{bmatrix}$$

4+3=7

12. *(a)* If $\phi = x^3 + y^3 + z^3 - 3xyz$, then find *(i)* div (grad ϕ) and *(ii)* curl (grad ϕ). 2+2=4

(b) If A be a square complex matrix, then show that it can be expressed as sum of a Hermitian and skew-Hermitian matrices. 3

★ ★ ★